

Atmospheric MUons from PArametric formulas: a fast GEnerator for neutrino telescopes (MUPAGE)

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Abstract

Neutrino telescopes will open, in the next years, new opportunities in observational high energy astrophysics. In these detectors, atmospheric muons from primary cosmic ray interactions in the atmosphere play an important role, because they provide the most abundant source of events for calibration and test. On the other side, they represent the major background source.

In this paper a fast Monte Carlo generator (called MUPAGE) of bundles of atmospheric muons for underwater/ice neutrino telescopes is presented. MUPAGE is based on parametric formulas [1] obtained from a full Monte Carlo simulation of cosmic ray showers generating muons in bundle, which are propagated down to 5 km w.e. It produces the event kinematics on the surface of a user-defined cylinder, surrounding the virtual detector. The multiplicity of the muons in the bundle, the muon lateral distribution and energy spectrum are simulated according to a specific model of primary cosmic ray flux, with constraints from measurements of the muon flux with underground experiments. As an example of application, the result of the generation of events on a cylindrical surface of $\sim 1.4 \text{ km}^2$ at a depth of 2450 m of water is presented.

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Key words: Simulation of atmospheric muons; neutrino telescopes; multiple muons; Monte Carlo generator.

PROGRAM SUMMARY

Title of program: MUPAGE

Nature of the physics problem: Fast simulation of atmospheric muon bundles for

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underwater/ice neutrino telescopes.

Method of solution: Atmospheric muon events are generated according to parametric formulas [1] giving the flux, the multiplicity, the radial distribution and the energy spectrum.

Licensing provisions: none.

Programming language used: C++

Computers on which the program has been tested: Pentium M, 2.0 GHz; 2x Intel Xeon Quad Core, 2.33 GHz

Number of processors used: one

Operating systems under which the program has been tested: Scientific Linux 3.x; Scientific Linux 4.x; Slackware 12.0.0

Number of bits in a word: 32

Memory required to execute program with typical data: 50 MB

Has the code been vectorized or parallelized? no

Number of lines in distributed program: 1 933

Number of bytes in distributed program: 59 166

Distribution format: tar

Library used: ROOT [26]

Additional comments: The program requires the ROOT libraries for the pseudorandom number generator.

Restrictions of the program: Water vertical depth range from 1.5 to 5 km w.e.; zenith angle range from 0 to 85 degrees.

Keywords: Simulation of atmospheric muons, neutrino telescopes, multiple muons, Monte Carlo generator

PACS: 95.55.Vj

Classification: 1.1 Cosmic Rays; 11.3 cascade and shower simulation.

1 Introduction

Most astrophysical sources are expected to produce neutrinos in addition to photons and cosmic rays [2]. Theoretical predictions for neutrino fluxes indicate that $\sim 1 \text{ km}^3$ scale apparatus is probably needed. A neutrino telescope in the South Pole is currently under construction and is taking data with a reduced configuration [3]. In the Northern hemisphere, the European consortium KM3NeT [4], including the three collaborations, ANTARES [5], NEMO [6] and NESTOR [7], is working on a design study for a large deep-sea neutrino telescope in the Mediterranean sea.

Although neutrino telescopes are located under large depth of water or ice, a great number of high energy atmospheric muons can reach the active volume of the detectors. Atmospheric muons are produced in the decay of charged mesons generated by the interactions of primary cosmic rays with atmospheric nuclei. They represent the most abundant signal in a neutrino telescope and they can be used to calibrate the detector and to check its response to the passage of charged particles. On the other side, they can constitute the major background source, mainly because downward-going muons can incorrectly be reconstructed as upward-going particles mimicking high energy neutrino interactions. Muons in bundles seem to be particularly dangerous [8]. A full Monte Carlo (MC) simulation, starting from the generation of atmospheric showers, can accurately reproduce the main features of muons reaching a neutrino telescope, but it requires a large amount of CPU time.

In this paper an event generator (MUPAGE: MUon GEnerator from Parametric formulas) based on parametric formulas [1] is presented. The formulas allow to calculate the flux and angular distribution of underwater/ice muon bundles, taking into account the muon multiplicity and the multi-parameter dependent energy spectrum. The range of validity extends from 1.5 km to 5.0 km of water vertical depth, and from 0° up to 85° for the zenith angle. MUPAGE output is an ASCII table containing the kinematics of events at the surface of a *can*. The *can* is an imaginary cylinder surrounding the active volume of a generic detector. The ASCII table can be used as input in the following steps of a detector-dependent MC simulation, which include production of light in water/ice and simulation of the signal in the detection devices.

This paper is organized as follows. In Section 2 the main features of the underwater muon flux, described by the parameterizations of [1], are reviewed. In Section 3 the MUPAGE structure is presented, in particular details are given on how to run the program, the input parameters and the output file format. Section 4 describes the generation of events on the surface of the *can*, with the use of a *Hit-or-Miss* method. Events can be single muons (Section 5) or the more complex multiple muons (Section 6). MUPAGE produces events with

the same relative weight. For each run the livetime is computed as described in Section 7. Finally, as an example of application, the case of the ANTARES detector is presented in Section 8.

2 Simulation of bundles of atmospheric muons

Atmospheric muon flux in the deep water is at present experimentally poorly known [9]. Some parameterisations for the underwater flux and energy spectrum are available in literature [10,11,12]. None of them gives the possibility to simulate two or more muons produced in the same cosmic ray interaction (muon bundles). To overcome this limitation, at a price of huge CPU time requirement, full simulations of atmospheric showers induced by primary cosmic ray interactions are performed by neutrino telescope collaborations [13,14]. These full MCs are generally based on the CORSIKA [15] or HEMAS [16] packages. They are then followed by the propagation of the surviving muons from sea level to the detector position.

MUPAGE relies on parametric formulas which describe the flux, the angular distribution and the energy spectrum of muon bundles for vertical depths h from 1.5 to 5 km w.e. of water or ice. The muon energy depends on h , on the zenith angle ϑ and, for muon in bundles, on bundle multiplicity m and on the distance R of each muon with respect to the shower axis. The parametric formulas were obtained starting from a full MC simulation of primary cosmic ray interactions and shower propagation in the atmosphere using the HEMAS code. HEMAS was preferred since it was largely used and cross-checked with MACRO data, a large underground experiment operating from 1994 to 2000 and located at the INFN Gran Sasso laboratory, at a depth comparable to that of neutrino telescopes. In particular, MACRO measured the flux of muon bundles [17] and the distribution of distances between muon pairs in a muon bundle (the so-called decoherence distribution [18]). The input primary cosmic ray spectrum used in MUPAGE was an unpublished MACRO model bounded by the measurements of underground muons. To optimize the full MC simulation, the code was restricted to follow secondary particles with energies $E > 500$ GeV. Muons reaching the sea level with energies $E > 500$ GeV were propagated through water down to 5 km using the MUSIC (MUon SIMulation Code) program [19]. The so called *prompt muons* (originated from the decay of charmed mesons and other short-lived particles) were not included. They were expected to give a not negligible contribution for muon energies from ~ 10 TeV to $\sim 10^3$ TeV, depending on the charm production model [20].

A MUPAGE event is a bundle of muons with multiplicity m_c on the *can*. Muons in the bundle are assumed to be parallel to the shower axis, and to reach at the same time a plane perpendicular to the shower axis. The bundle

multiplicity, direction and impact point of the shower axis on the *can* surface are generated first. Then, for each muon in the bundle, the distance from the shower axis, the energy and the coordinates of the impact point on the *can* surface are calculated.

2.1 Parametric formulas for the flux, energy spectrum and radial distance of muons

The flux of bundles with muon multiplicity m at a given vertical depth h and zenith angle ϑ is parameterized using two free parameters (K and ν) as:

$$\Phi(h, \vartheta, m) = \frac{K(h, \vartheta)}{m^{\nu(h, \vartheta)}} \quad (1)$$

As for the following formulas in this subsection, more details on the functional dependence of the parameters are reported in [1]. Fig. 1 shows the flux of muon bundles with different multiplicity at the depth $h = 2.5$ km w.e. for five different zenith angles, $\vartheta = 0^\circ, 20^\circ, 40^\circ, 60^\circ$ and 70° obtained using (1).

Fig. 2 shows the flux of muons (with energy $E_\mu > 20$ GeV) versus the zenith angle cosine at three different depths. In this case, a bundle of multiplicity m is accounted for m muons. The full lines are obtained with (1), for depths $h = 1.64, 2.0$ and 3.0 km w.e. The points represent the AMANDA-II unfolded measurement [21], at the depth $h = 1.64$ km w.e. and with the same muon energy threshold. The (red) dotted lines are from [12], at depths $h = 1.61, 2.0$ and 3.0 km w.e. In [12], the bundle multiplicity was not taken into account and the depth-intensity relation of Bugaev et al. [11] was used as input. In our case, the depth-intensity relation for the vertical direction (called in literature $I(h, 0)$) is computed with (1) and compared with other parameterizations in Fig. 1 of [1]. The (normalized) difference between what obtained with (1) and what reported in [11] is -16% at 1.5 km w.e. and -5% at 5.0 km w.e. The differences at the same depths of (1) with respect to the Okada parameterization [10] are +1% and -7%, respectively.

The expected energy distribution of muons, assuming a power-law for the primary beam energy, at a slant depth $X = h/\cos\vartheta$ is [22]:

$$\frac{dN}{d(\log_{10} E_\mu)} = G \cdot E_\mu e^{\beta X(1-\gamma)} [E_\mu + \epsilon(1 - e^{-\beta X})]^{-\gamma} \quad (2)$$

The quantities ϵ and γ are considered as free parameters, while β as a constant. $G = G(\gamma, \epsilon)$ represents a normalization factor, in order to get the integrated

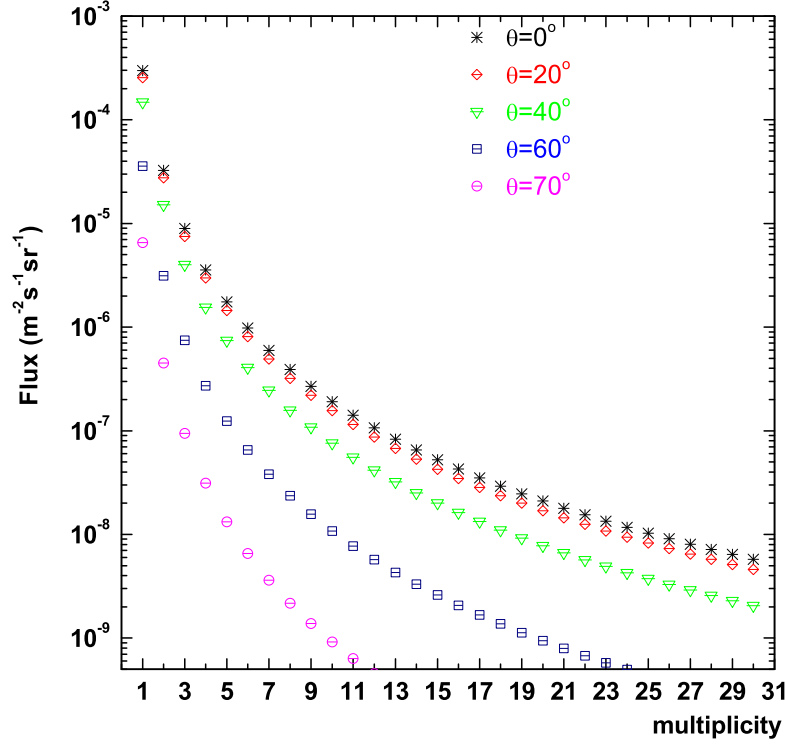


Fig. 1. Flux of muon bundles with different multiplicity m at the depth of $h = 2.5$ km w.e. for five different zenith angles, $\vartheta = 0^\circ, 20^\circ, 40^\circ, 60^\circ$ and 70° as computed with (1).

muon energy spectrum up to 5×10^5 GeV equal to 1. MUPAGE uses (2) to extract the muon energy, both for single muon events and for muon bundles.

- **Single muons (bundles with multiplicity $m = 1$).** The parameter γ depends on the vertical depth h only: $\gamma = \gamma(h)$. The parameter ϵ depends both on h and on the zenith angle ϑ : $\epsilon = \epsilon(h, \vartheta)$. See Section 5.
- **Multiple muons (bundles with multiplicity $m > 1$).** Muons produced in the decay of secondary mesons follow the energy distribution of the parent mesons. As a consequence, the most energetic muons are expected to be closer to the axis shower. To sample the energy of a muon in a bundle, its radial distance R from the shower axis is taken into account. The muon radial distance distribution is described as:

$$\frac{dN}{dR} = C \frac{R}{(R + R_0)^\alpha} \quad (3)$$

where the parameter R_0 depends on the vertical depth h , on the bundle multiplicity m and on the zenith angle ϑ : $R_0 = R_0(h, m, \vartheta)$; α depends on h and m : $\alpha = \alpha(h, m)$. Finally, $C = C(R_0, \alpha)$ is a normalization factor.

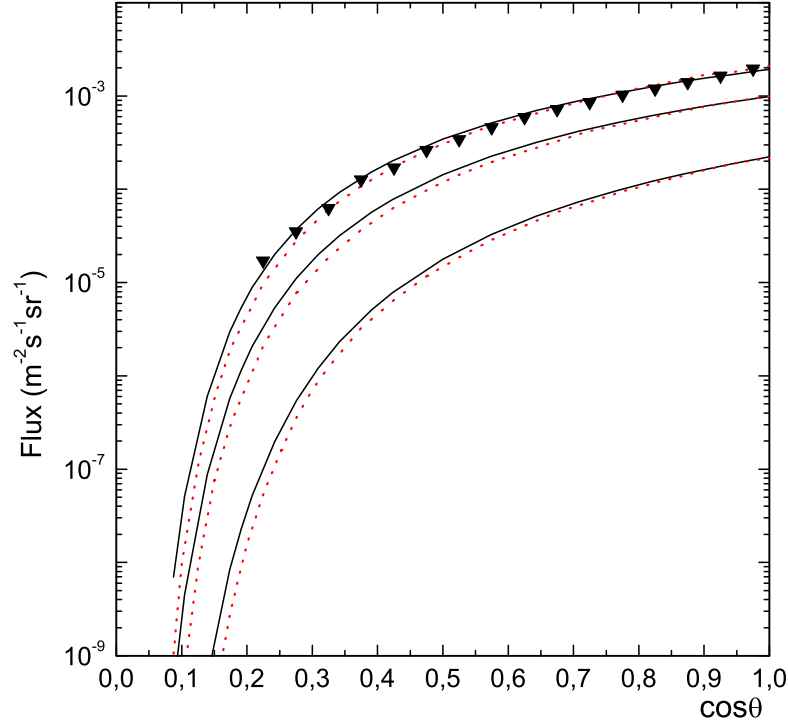


Fig. 2. Muon flux (with energy $E_\mu > 20$ GeV) as a function of the cosine of the zenith angle and for three different depths. The full lines are obtained from Eq. (1), for depths $h = 1.64, 2.0$ and 3.0 km w.e. from top to bottom. The points represent the AMANDA-II measurement [21], at the depths $h = 1.64$ km w.e. and with the same muon energy threshold. The (red) dotted lines are from [12], at depth $h = 1.61, 2.0$ and 3.0 km w.e.

For each muon in the bundle, the energy is extracted with a probability given by the distribution (2). In this case, the parameter γ depends on the vertical depth h , on the bundle multiplicity m and on the radial distance R : $\gamma = \gamma(h, m, R)$. The parameter ϵ depends on h , on the zenith angle ϑ , and on R : $\epsilon = \epsilon(h, \vartheta, R)$. See Section 6.

Eqs. (1), (2) and (3) are implemented in the MUPAGE `Muons` class as described in the next section. The values of the numerical constants are implemented in the `Parameters` class.

3 Program structure

MUPAGE code is written in C++ and it has been tested with gcc version 3.2.x, 3.4.x and 4.1.x. The program requires ROOT libraries [26] for the pseu-

File name	File type	Contents
Makefile	make file	file to create the executable
run-mupage.csh	C-Shell script file	template script (tcsh)
run-mupage.sh	Shell script file	template script (bash)
README	ASCII file	instructions on running MUPAGE
dat	directory	input parameter files (see Section 3.1)
evt	directory	event output files (see Section 3.2)
inc	directory	C++ include files for MUPAGE
livetime	directory	run livetime (see Section 7)
src	directory	MUPAGE C++ source code

Table 1

Description of the files contained in the *tar* file main directory **mupage/**

dorandom number generator (see Section 4.1). Files from the *tar* archive are extracted in the main folder **mupage/**. This folder contains the **Makefile**, a **README** ASCII file, two template script files (for tcsh and for bash) to launch the executable and some subdirectories as described in Table 1. After running **Makefile**, a **Linux/** directory is created, containing the object files; the executable is created in the main directory. The prototypes files (**.hh**) are in the **inc/** directory and the declaration files (**.cc**) in the **src/** directory. They do not need to be changed by the user:

inc/ConvertingUnits.hh :	units conversion
inc/Geometry.hh :	geometry definition
inc/Parameters.hh & src/Parameters.cc:	class with parameters from [1] to compute the parametric formulas
inc/Decode.hh & src/Decode.cc :	class to decode input parameters
inc/Muons.hh & src/Muons.cc :	class to generate an event
src/mupage.cc :	main program

The simplest way to execute MUPAGE is the use of the C-shell script file **run-mupage.csh** for tcsh, or the Shell script file **run-mupage.sh** for bash. In both template scripts the user can modify random seed, run number and the number N_{gen} of events to be generated. It is also possible to define the directories for MUPAGE output files. The livetime of the run is evaluated from N_{gen} (see Section 7). The following control cards, defined in the template scripts, can be set as **-character [name of data]**:

-h	help
-i \$run_id	run number
-n \$num_events	number of events to be generated
-s \$seed	random seed (default 0)
-p \$INFILE	full name (including path) of the input file (see Section 3.1)
-o \$OUTFILE1	full name (including path) of the first output file
\$OUTFILE2	full name (including path) of the second output file (for both output files see Section 3.2)

Card values are provided as arguments to the executable:

```
./mupage.exe -i $run_id -n $num_events -s $seed -p $INFILE
-o $OUTFILE1 $OUTFILE2
```

3.1 Description of the input file

The generator needs some parameters as input, which are contained in the file **parameters.dat** in the directory **dat/**. Parameters refer to the detector configuration (see Fig. 3), to the medium and to the range of simulation parameters (multiplicity, zenith angle, muon energy). Name, default value and unit of parameters are described in Table 2².

Note that, in the input file, the **MULTmax** parameter has a default value equal to 1000. This is the only parameter affecting CPU time significantly. To optimize CPU time, the event files can be produced with different ranges of [**MULTmin**, **MULTmax**]. In this case, when filling histograms, results must be normalized taking into account different values of livetime for each file (see Section 8) .

3.2 Description of the output files

The code provides two output data files: **\$OUTFILE1** e **\$OUTFILE2**. The former is written (by default) in the directory **mupage/evt** and contains all information about the generated events in a formatted ASCII table. An example of output data file with 1000 generated events (**run_01.evt**) is present

² Default values refer to the ANTARES experiment in the Mediterranean sea, see Section 8.

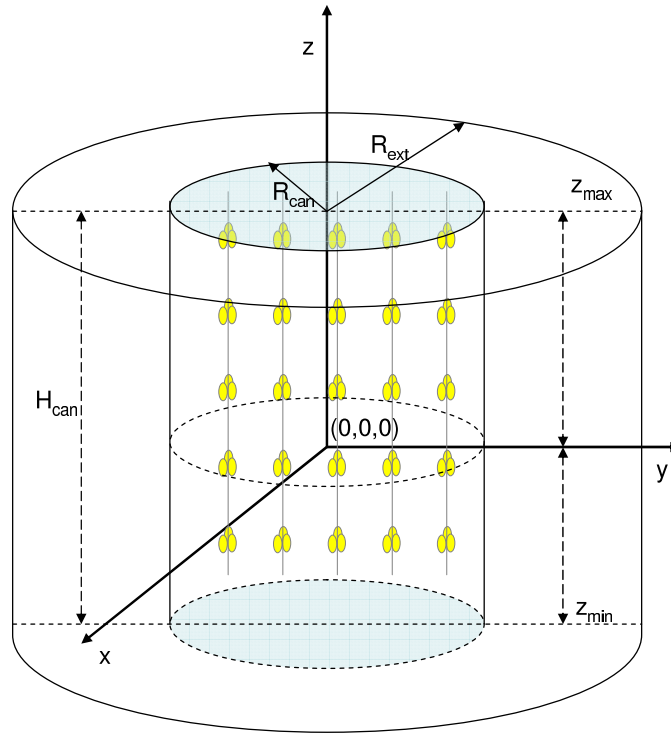


Fig. 3. Sketch of the geometrical meaning of some input parameters. The cylinder surrounding the instrumented volume is the *can*, with radius R_{can} and height H_{can} . The events are generated on an extended can, with $R_{ext} = R_{can} + R_{ecr}$. The origin of coordinate system lies on the cylinder axis, but not necessarily at the center of the cylinder. The lower disk is at a depth H_{max} with respect to the sea/ice surface. The origin of the detector coordinate system lies on the cylinder axis, but it does not necessarily coincide with the center of the cylinder.

in the folder. Each line of the table contains the following information:

event_id mult track_id x_i y_i z_i v_x v_y v_z E_i t_i μ_id

where:

- **event_id** = event number;
- **mult** = multiplicity of the muon bundle at the depth where the shower axis hits the *can*;
- for each muon in the bundle and intercepting the *can* ($m_c \leq mult$):
 - $track_id = i$ ($i = 1, m_c$), muon identifier in the event;
 - (x_i, y_i, z_i) , coordinates of the muon impact point on the can surface;
 - (v_x, v_y, v_z) , direction cosines of the muon, coincident with those of the bundle axis;
 - E_i (in GeV), energy of the muon;
 - t_i , time delay of the i -th muon at the *can* surface with respect to the first muon in the list, ($i = 1$). t_i can be either positive or negative;
 - μ_id , muon particle identification number, for the detector-dependent simulations following the MUPAGE step. By default it is inserted the GEANT3

Parameter	Default value	Description
Hmax (H_{max})	2.475 km	vertical height of the sea/ice level with respect to the <i>can</i> lower disk
Zmin (Z_{min})	-278.151 m	minimum z-coordinate of the <i>can</i> w.r.t. the center of gravity system of detector
Zmax (Z_{max})	313.971 m	maximum z-coordinate of the <i>can</i> w.r.t. the center of gravity system of detector
CANr (R_{can})	238.611 m	<i>can</i> radius
EnlargedCANr (R_{ecr})	300.0 m	increase of the <i>can</i> radius
density	1.025 g cm ⁻³	mean value of medium density
AbsLength (λ_{abs})	55.0 m	medium absorption length
THET Amin (ϑ_{min})	0.0°	minimum zenith angle of the bundle
THET Amax (ϑ_{max})	85.0°	maximum zenith angle of the bundle
Rmin (R_{min})	0.0 m	minimum lateral spread of multiple muons
Rmax (R_{max})	100.0 m	maximum lateral spread of multiple muons
Emin (E_{min})	0.02 TeV	minimum muon energy at the <i>can</i> level
Emax (E_{max})	500.0 TeV	maximum muon energy at the <i>can</i> level
Ethreshold (E_{thr})	0.02 TeV	threshold energy = $\sum_{i=1}^{m_c} E_{\mu,i}$
MULTmin (m_{min})	1	minimum muon multiplicity
MULTmax (m_{max})	1000	maximum muon multiplicity
GEANTid	6	muon ID (default: the GEANT3 id [23])
MFactor	1.0	multiplicative factor for special geometries

Table 2

Description and default values of parameters in the input file `/dat/parameters.dat`.

code μ^- =6. GEANT4 uses the Particle Data Group encoding and μ^- =13 [23]. By inserting the appropriate ID number for the muon, any other MC can be used.

A second output file **\$OUTFILE2** is written (by default) in the subdirectory `mupage/livetime` and contains the livetime (units: seconds and days) for the simulated run. The livetime is given with its computed statistical error. The evaluation of livetime is described in Section 7. All events have the same weight (=1). The flowchart of the Monte Carlo program is shown in Fig. 4; details are described in Sections 4, 5 and 6.

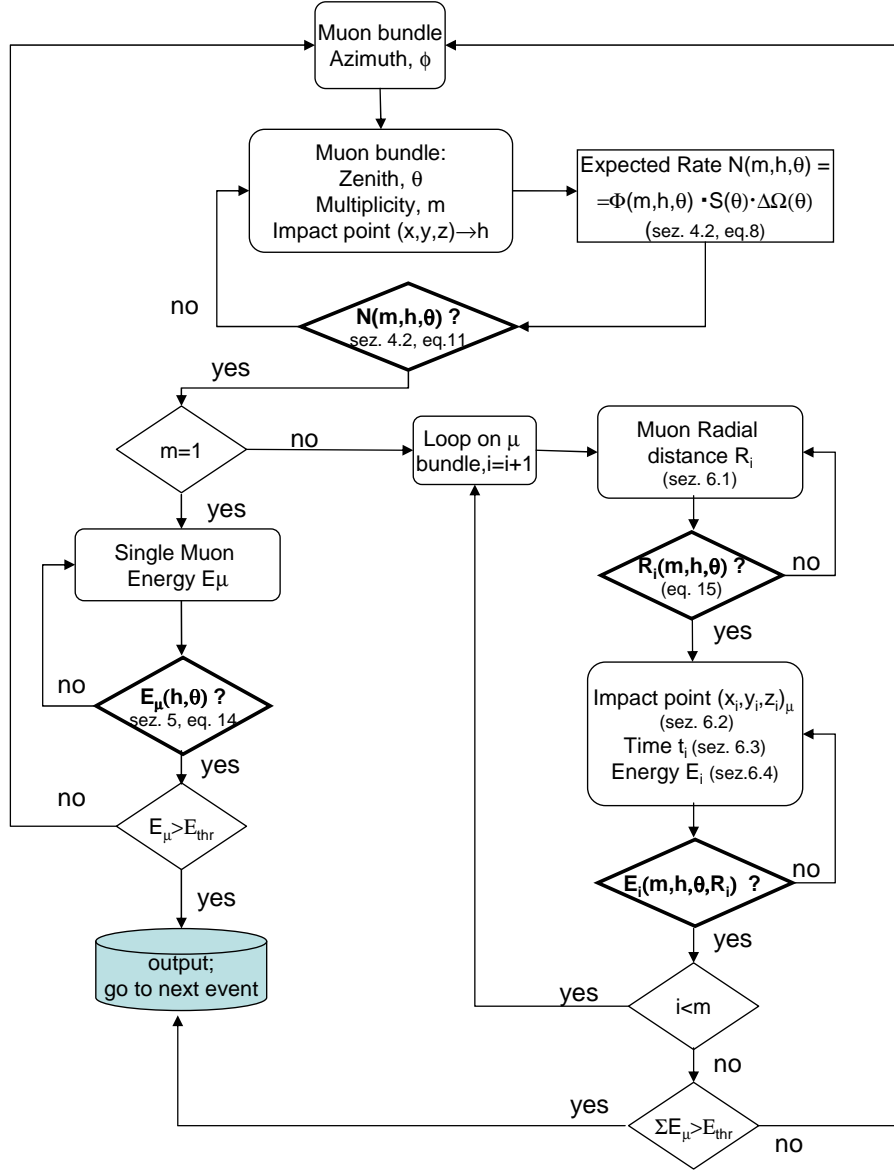


Fig. 4. The flowchart of the MUPAGE event generator. The smooth-angle rectangles indicate the extraction of uniformly distributed random values. The decisional rhombuses in bold select values according to formulas reported in Section 2, with a *Hit-or-Miss* method. The procedure is iterated for N_{gen} events.

4 Generation of muon bundles on the *can* surface

4.1 Sampling the bundle direction and impact point on the *can* surface

Muon bundles at ~ 2 km water equivalent depth can have multiplicity as large as 10^3 , and muons can be hundreds of meters far from the axis shower. On

the other hand, muons travelling few absorption lengths far from the detector have a small probability to give a signal on photomultipliers. In order to accept peripherals muons in large bundles, the radius of the generation volume is increased by a quantity **EnlargedCANr** (R_{ecr}), specified in data cards. The radius of the generation cylinder is $R_{ext} = R_{can} + R_{ecr}$. If R_{can} coincides with the radius of the cylinder surrounding the instrumented volume of the detector, it is recommended to define $R_{ecr} \simeq 10\lambda_{abs}$.

As a first step, a generic bundle with muon multiplicity $m^* \in [m_{min}, m_{max}]$, random zenith angle $\vartheta^* \in [\vartheta_{min}, \vartheta_{max}]$ and azimuth angle $\phi^* \in [0^\circ, 360^\circ]$ in the detector frame is generated. The values that are extracted from uniform distributions, as m, ϑ and ϕ at this step, are denoted with a *. The pseudorandom number generator used in the program is the Mersenne Twister algorithm [25] and it is included in the ROOT libraries (TRandom3 class) [26]. The axis of the bundle with (ϑ^*, ϕ^*) intercepts the extended *can* in a random point of coordinates (x^*, y^*, z^*) , which are computed in the following way. The cylinder projected area seen by the bundle is shown in Fig. 5. The impact point (X_R, Y_R) is a random point on this plane. Only downward going particles are generated. It means that points with $z^* = Z_{min}$, on the lower disk of the *can* are not considered. L_x and L_y in Fig. 5 are defined as $L_x = 2R_{ext}$ and $L_y = H_{can} \sin \vartheta^* + 2R_{ext} \cos \vartheta^*$. The coordinates X_R and Y_R can assume the values:

$$-R_{ext} \leq X_R \leq R_{ext} \quad (4)$$

$$-\left(\frac{H_{can}}{2} \sin \vartheta^* + R_{ext} \cos \vartheta^*\right) \leq Y_R \leq \frac{H_{can}}{2} \sin \vartheta^* + R_{ext} \cos \vartheta^* \quad (5)$$

The point (X_R, Y_R) on the plane perpendicular to the shower direction can be on the upper disk or on the lateral surface of the *can*. It lies on the upper disk (grey area of Fig. 5) if:

$$Y_R > \frac{H_{can}}{2} \sin \vartheta^* - R_{ext} \cos \vartheta^* \quad (6)$$

and

$$\frac{X_R^2}{R_{ext}^2} + \frac{[Y_R - (H_{can}/2) \sin \vartheta^*]^2}{(R_{ext} \cos \vartheta^*)^2} \leq 1 \quad (7)$$

Returning in 3-D, the point coordinates are: (x^*, y^*, z^*) , with $x^* = -X_R$, $y^* = \frac{Y_R}{\cos \vartheta^*} - \frac{H_{can}}{2} \tan \vartheta^*$ and $z^* = Z_{max}$.

If Eqs. (6), (7) are not both true, the shower axis hits the lateral surface.

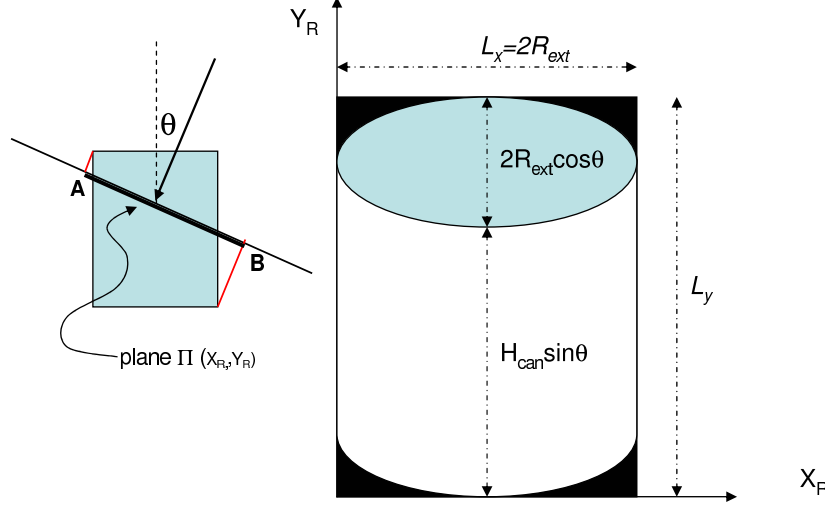


Fig. 5. Left: sketch of the plane Π perpendicular to the shower axis. The muon bundle has zenith angle ϑ with respect to the detector z -axis. The interception point of the shower axis is uniformly distributed on the cylinder projection on the Π plane. They will be generated outside the black region; the events in the grey area lie in the upper disk of the cylinder, while the remaining in the lateral surface.

In this case, the intersection point has coordinates $x^* = R_{ext} \cos \phi'$, $y^* = R_{ext} \sin \phi'$, $z^* = \frac{Y_R + \cos \vartheta^* \sqrt{R_{ext}^2 - X_R^2}}{\sin \vartheta^*} + \frac{Z_{min} + Z_{max}}{2}$ where $\phi' = \phi^* + \frac{3}{2}\pi - \arccos\left(-\frac{X_R}{R_{ext}}\right)$. The points (x^*, y^*, z^*) are distributed uniformly on the *can* surface (with the exclusion of the lower disk).

4.2 Hit-or-Miss method to sample the impact point

The flux $\Phi(h, \vartheta, m)$, Eq. (1), decreases with increasing depth h , zenith angle ϑ and muon multiplicity m , as shown in Figs. 1 and 2. The procedure described in Section 4.1 extracts uniformly h^* , ϑ^* and m^* . A *Hit-or-Miss* method [24] is used to reproduce the correct dependence of the number of events on these variables. For each set of parameters (h^*, ϑ^*, m^*) , the number of events arriving on the projected area $S(\vartheta^*)$ in a small solid angle $\Delta\Omega(\vartheta^*)$ centred around ϑ^* is computed:

$$N_{proj}(h^*, \vartheta^*, m^*) = \Phi(h^*, \vartheta^*, m^*) \cdot S(\vartheta^*) \cdot \Delta\Omega(\vartheta^*) \quad (8)$$

where

$$S(\vartheta^*) = \pi R_{ext}^2 \cdot \cos \vartheta^* + 2R_{ext}H_{can} \cdot \sin \vartheta^* \quad (9)$$

$$\Delta\Omega(\vartheta^*) = 2\pi[\cos(\vartheta^* - 0.5^\circ) - \cos(\vartheta^* + 0.5^\circ)] \quad (10)$$

A random number u is then generated, with:

$$0 < u < N_{max} \simeq \Phi(H_{min}, \vartheta_{min}, m_{min}) \cdot S(\vartheta') \cdot \Delta\Omega(\vartheta') \quad (11)$$

N_{max} corresponds to the set of values (h, ϑ, m) for which the function $\Phi(h, \vartheta, m) \cdot S(\vartheta) \cdot \Delta\Omega(\vartheta)$ is maximum. This function has a maximum for the minimum value of the detector depth ($h = H_{min}$), corresponding to the *can* upper disk, and for the minimum value of the range of muon multiplicities ($m = m_{min}$). The maximization in terms of the ϑ variable is more complex, due to the not trivial dependence of $\Phi(h, \vartheta, m) \cdot S(\vartheta) \cdot \Delta\Omega(\vartheta)$ on ϑ .

In order to save CPU time, the maximum of $\Phi(H_{min}, \vartheta, m_{min}) \cdot S(\vartheta) \cdot \Delta\Omega(\vartheta)$ is computed as the product of the maximum of the functions $\Phi(H_{min}, \vartheta, m_{min})$ and $S(\vartheta) \cdot \Delta\Omega(\vartheta)$. The former has a maximum in correspondence of ϑ_{min} . The latter has a maximum, from (9) and (10), for zenith angle $\vartheta' = \arctan\left(\frac{\pi R_{ext}}{2H_{can}}\right)$. Using this approximation N_{max} is evaluated as in (11). The parameter set (h^*, ϑ^*, m^*) is accepted if:

$$u < N_{proj}(h^*, \vartheta^*, m^*) \quad (12)$$

In the following, to simplify the notation, (h, ϑ, m) will be used and the impact point coordinates become (x, y, z) .

If the *can* height is much larger than the disk diameter, $H_{can} \gg R_{ext}$, the approximation used for N_{max} in (11) is not valid. It must be stressed that this is NOT the case for present neutrino telescope configurations. However, to correct the procedure, a multiplicative factor in the input parameters is introduced (default value =1). When the error message appears:

ERROR! Nmax must be larger than Nproj

and the program stops, the user must change **MFactor** in the input data file (parameters.dat) to a value larger than 1 (usually **MFactor** ~ 1.2 is enough). As an alternative, the value of R_{ecr} can be increased.

5 Single muons

The underwater/ice flux of atmospheric muons is dominated (Fig. 1) by events reaching the detector with multiplicity $m = 1$, the so-called *single muons*. In this case the muon direction is assumed coincident with the shower axis and the impact point is (x, y, z) . The arrival time of the muon on the *can* surface is $t = 0$. The muon energy E is extracted according to (2), whose parameters depend on the vertical depth h of the impact point on the *can* surface and on the muon zenith angle ϑ .

A value of $\log_{10} E^*$ is generated randomly between $\log_{10} E_{min}$ and $\log_{10} E_{max}$ (E_{min}, E_{max} are given in the data cards). The value E^* is accepted (or rejected) according to the *Hit-or-Miss* method: a random number u' is generated between 0 and the maximum of (2) at depth h and zenith ϑ :

$$0 < u' < \left(\frac{dN}{d(\log_{10} E_\mu)} \right) (h, \vartheta; E_\mu^{max}) \quad (13)$$

The maximum of (2) occurs in correspondence of $E_\mu^{max} = \frac{\epsilon(1-e^{-\beta X})}{(\gamma-1)}$. The value E^* is accepted if:

$$u' < \frac{dN}{d(\log_{10} E_\mu)} (h, \vartheta; E^*) \quad (14)$$

6 Multiple muons

6.1 The radial distance of muons with respect to the bundle axis

For events with muon multiplicity $m > 1$, the distance R of each muon from the bundle axis (in a plane perpendicular to the axis) is calculated, according to the radial distribution (3). R depends on depth h , on bundle multiplicity m and on the zenith angle ϑ . It is useful to define a new reference frame (*Bundle Axis Frame*, *BAFrame*), where the z_{BAF} -axis coincides with the axis shower, see Fig. 6. Each muon is located in a point (X, Y) of the plane Π perpendicular to the axis shower. The distance of the point (X, Y) from the origin of the *BAFrame* is called R_i (the muon radial distance). R_i^* is sampled randomly between R_{min} and R_{max} (both values from data cards). The *Hit-or-Miss* method is used to accept (or reject) the value R_i^* . A random number u'' is generated between 0 and the maximum of the lateral distribution function

(dN/dR) at the given h , ϑ and m . R_i^* is accepted if:

$$u'' < \frac{dN}{dR}(h, \vartheta, m, R_i^*) \quad (15)$$

The coordinates in the *BAFrame* are computed from the selected R_i as $X = R_i \cdot \cos \beta$ and $Y = R_i \cdot \sin \beta$, where β is a random number between 0 and 2π .

6.2 Coordinates of the multiple muons on the can surface

The shower axis intercepts the *can* in the impact point (computed in Section 4.2) with coordinates $(x, y, z) = (x_{SA}, y_{SA}, z_{SA})$. Then, referring to Fig. 6, for each muon in the bundle:

- (X, Y) = coordinates of the muon in the *BAFrame*;
- (x_μ, y_μ, z_μ) = coordinates of the muon in the laboratory frame;
- $(v_x, v_y, v_z) = (\sin \vartheta \cos \phi, \sin \vartheta \sin \phi, \cos \vartheta)$ = direction of the muon in the laboratory frame;
- (x_i, y_i, z_i) = projection of the point (x_μ, y_μ, z_μ) along the shower direction on the *can* surface.

When the point (X, Y) in the *BAFrame* is known, the point (x_μ, y_μ, z_μ) in the laboratory frame can be computed using a general matrix \mathbf{A} resulting from the composition of three rotations [27]. In the so-called ‘X-convention’ the rotations are defined by the Euler angles (Φ, Θ, Ψ) , where the first rotation is by an angle Φ around the z-axis, the second one is by an angle $\Theta \in [0, \pi]$ around the x-axis and the third one is by an angle Ψ around the z-axis (again). There is a univocal relationship between the three Euler angles and the zenith (ϑ) and azimuth (ϕ) angles:

$$\Phi = -\pi/2, \Theta = \vartheta \text{ and } \Psi = \phi + \pi/2.$$

The transformation of the point (X, Y) in the *BAFrame* into the point (x_μ, y_μ, z_μ) in the laboratory frame is defined as:

$$\begin{pmatrix} x_\mu \\ y_\mu \\ z_\mu \end{pmatrix} = \mathbf{A} \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix} \quad (16)$$

The coordinates of the impact point of each muon on the *can* are obtained using the projection of each point (x_μ, y_μ, z_μ) along the direction (v_x, v_y, v_z) . This is done using the straight line defined by the three parametric equations

$(x_i, y_i, z_i) = (x_\mu, y_\mu, z_\mu) + k(v_x, v_y, v_z)$, where $k = (Z_{max} - z_\mu)/v_z$. The impact point of the i -th muon in the bundle is on the upper disk of the *can* if the straight line intercepts the plane $z = Z_{max}$ with $x_i^2 + y_i^2 \leq R_{ext}^2$. In this case, the coordinates are:

$$(x_i, y_i, z_i) = (x_\mu + kv_x, y_\mu + kv_y, Z_{max}) \quad (17)$$

If $x_i^2 + y_i^2 > R_{ext}^2$, the impact point (x_i, y_i, z_i) lies on the *can* lateral surface or it does not intercept the *can* at all.

The intersection of the straight line with the lateral surface of the *can* (defined by equation $x^2 + y^2 = R_{ext}^2$) gives a second degree equation $a\Lambda^2 + 2b\Lambda + c = 0$, with $a = v_x^2 + v_y^2$, $b = v_x x_\mu + v_y y_\mu$ and $c = x_\mu^2 + y_\mu^2 - R_{ext}^2$. The solutions are $\Lambda_\pm = \frac{-b \pm \sqrt{\Delta}}{a}$, with $\Delta = b^2 - ac$. If $\Delta < 0$ the i -th muon does not intercept the *can*. For $\Delta \geq 0$ the two possible impact points are:

$$(x_i, y_i, z_i) = (x_\mu + \Lambda_+ v_x, y_\mu + \Lambda_+ v_y, z_\mu + \Lambda_+ v_z) \quad (18)$$

$$(x_i, y_i, z_i) = (x_\mu + \Lambda_- v_x, y_\mu + \Lambda_- v_y, z_\mu + \Lambda_- v_z) \quad (19)$$

As atmospheric muons are downward going, the solution with the larger value z_i is chosen. If $z_i < Z_{min}$ the i -th muon does not intercept the *can*. The number of muons intercepting the *can* surface determines the bundle multiplicity $m_c \leq m$ at the *can*.

6.3 Arrival time of the muons in the bundle

All muons in the bundle are assumed to arrive at the same time on the plane Π perpendicular to the shower axis. In general, each muon reaches the *can* surface at a different time. The distance between the impact point of the i -th muon $P_i(x_i, y_i, z_i)$ and the coordinates of that muon on the plane Π in the laboratory frame $P_\mu(x_\mu, y_\mu, z_\mu)$ is:

$$d(P_i, P_\mu) = \text{sign} \cdot \sqrt{(x_i - x_\mu)^2 + (y_i - y_\mu)^2 + (z_i - z_\mu)^2} \quad (20)$$

The arrival time of the first muon in the list ($i = 1$) on the *can* surface is taken as $t_1 = 0$. All the remaining muons, labelled with $i = 2, \dots, m_c$, can intercept the *can* earlier ($t_i < 0$) or later ($t_i > 0$). The relative time is computed from the distance $d(P_i, P_\mu)$ defined by (20). A distance is an always positive quantity, but the evaluation of the relative delay between muons in the bundle requires the definition of the sign in (20). Referring to Fig. 6, if $z_\mu < z_i$ the distance is

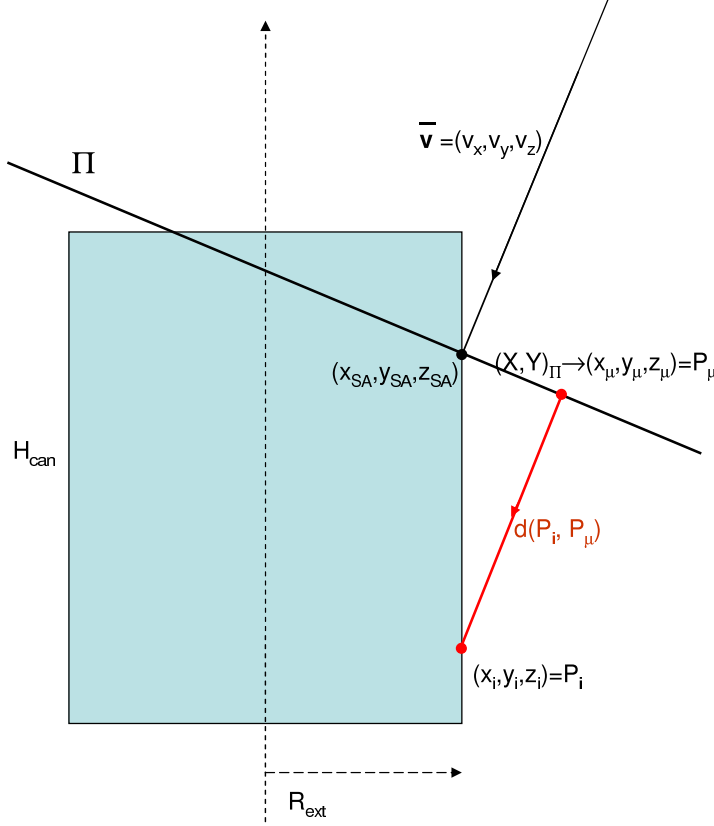


Fig. 6. Lateral view of the extended *can* (the rectangle). The shower axis has direction (v_x, v_y, v_z) and intercepts the detector in the point (x_{SA}, y_{SA}, z_{SA}) . m muons are generated on the plane Π perpendicular to the shower axis and $m_c \leq m$ intercept the *can*. A muon has coordinates (X, Y) in the *BAFrame* and (x_μ, y_μ, z_μ) in the laboratory frame. The point (x_i, y_i, z_i) is the projection of the point (x_μ, y_μ, z_μ) along the μ direction on the *can*. The time delay of each muon is evaluated from the distance between the points (x_i, y_i, z_i) and (x_μ, y_μ, z_μ) .

assumed positive ($sign = 1$), otherwise it is negative ($sign = -1$). The delay t_i of the i -th muon with respect to the first one is:

$$t_i = \frac{d(P_i, P_\mu) - d(P_1, P_{\mu 1})}{c} \quad (21)$$

where $c = 2.99 \times 10^8$ m/s is the light velocity. Since the distances can be either positive or negative, also t_i (in ns) will assume either positive or negative values.

6.4 Muon energy for multimMuon events

The last step is the choice of the energy of each muon in a multimMuon bundle according to the energy distribution (3). The muon energy extracted from this distribution depends on vertical depth, zenith angle, on the multiplicity of the shower and on the radial distance of the muon from the shower axis. The steps described in Section 5 for the energy of single muons are repeated for the evaluation of the energy of each muon in the bundle.

7 Livetime of the simulation

MUPAGE computes automatically the detector livetime corresponding to the number of generated events N_{gen} on the *can* surface. The number of simulated events $N(\Delta\Omega_i)$ in a small solid angle $\Delta\Omega_i = 2\pi(\cos\vartheta_{1i} - \cos\vartheta_{2i})$, with multiplicity $m = m_{min}$ and with shower axis intercepting the *can* upper disk, are evaluated in 33 bins³. The expected rate of muon events with multiplicity m_{min} on the *can* upper disk with area πR_{ext}^2 at the depth H_{min} , and in the solid angle $\Delta\Omega_i$ is:

$$\dot{N}_{MC}(\Delta\Omega_i) = \Phi(H_{min}, \vartheta_i, m_{min}) \cdot S \cdot \Delta\Omega_i \quad (s^{-1}) \quad (22)$$

where $\vartheta_i = (\vartheta_{1i} + \vartheta_{2i})/2$, and $S = \pi R_{ext}^2 \cos\vartheta_i$ (m^2) is the projected area of the upper disk. The equivalent livetime for each bin is:

$$T(\Delta\Omega_i) = N(\Delta\Omega_i) / \dot{N}_{MC}(\Delta\Omega_i) \quad (s) \quad (23)$$

The livetime with its statistical error is computed as the weighted average of the 33 values of $T(\Delta\Omega_i)$ (which have the same value, within statistical errors), and written in the **\$OUTFILE2** file.

8 An example of application: the case of ANTARES

The ANTARES (Astronomy with a Neutrino Telescope and Abyss environmental RESearch) collaboration is operating the largest neutrino telescope in the Northern hemisphere in a site 2475 m deep, 40 km off La-Seyne-sur-Mer

³ The first bin is $0^\circ < \vartheta < 10^\circ$, then 30 bins of 2 degrees are considered. The last two bins are $70^\circ < \vartheta < 76^\circ$ and $76^\circ < \vartheta < 85^\circ$, respectively. The bin size was chosen in order to have a constant or at least an adequate statistical sample in each bin.

(France). The detector (completed on May 2008) consists of an array of 12 lines separated one from each other on the seabed by 60-80 m. The instrumented area is $\sim 0.06 \text{ km}^2$ [5]. The simulation of atmospheric muons is one of the main task for ANTARES as for other neutrino telescopes. A full MC simulation is used, at the cost price of a large CPU consuming time. The ANTARES software tools are described in [13].

MUPAGE was used to produce a large sample of atmospheric muons. A data set with a livetime equivalent to one month of real data was generated with input parameters reported in Table 2. 358 files were created, each one smaller than 2 GB, with 10^7 events/file and corresponding to $7260 \pm 4 \text{ s}$. The CPU time required to produce a file (on a 2xIntel Xeon Quad Core, 2.33 GHz) was about 1 hour. It is a relatively large amount of CPU, but it is small when compared to other steps of simulation, namely the tracking and the Cherenkov light emission, which need ~ 10 times more CPU. Each file, after triggering and data conversion to a format equivalent to raw data, is equivalent to one run of 2.02 h of real data and it is used to test the detector response. The rate of generated events for a detector with parameters equal to the default value of Table 2 is 1240 Hz. This rate holds for muons above the threshold energy (20 GeV) at the surface of the generation cylinder of 1.4 km^2 and it is much larger (a factor almost 1000) than the actually observed event rate after applying a realistic trigger algorithm. To be triggered, muons must pass sufficiently close to the instrumented detector volume to produce hits on a minimum number of optical sensors. Analysis of the comparisons between the MUPAGE, the full MC simulation and the data is in progress [29].

A much larger data set of atmospheric muons is needed for background study of the high energy neutrino ($E_\nu > 100 \text{ TeV}$) diffuse flux [14,28]. In this case, it is not necessary to simulate the bulk of lower energy muon bundles. A data set equivalent to one year was generated, using an optimized choice of multiplicity ranges, and a very conservative cut on the energy threshold $E_{thr} = \sum_{i=1}^{m_c} E_{\mu,i} > 3 \text{ TeV}$ (set by the parameter **Ethreshold**, see Table 2). With $E_{thr} > 3 \text{ TeV}$, the rate of generated events on the default detector cylinder reduces to 27 Hz. The event multiplicity was divided in 6 sub ranges: $m = 1$ (4.7 Hz); $m = 2$ (4.4 Hz); $m = 3$ (3.1 Hz); $m = 4 \div 10$ (10.4 Hz); $m = 11 \div 100$ (4.2 Hz) and $m = 101 \div 1000$ (0.04 Hz). Each generated file with $m = 1, 2, 3, [4 \div 10]$ and $[11 \div 100]$ contains 10^6 events (in order to have size $< 2 \text{ GB}$). It respectively corresponding to 59, 63, 90, 29 and 62 hours of livetime. 5×10^4 events/file were generated for $m = [101 \div 1000]$ corresponding to a livetime of 350 hours/file. The total number of generated files is 852. The CPU time required to generate each file ranges from 10 to 30 minutes using the same processor quoted above (it increases with the value of the minimum multiplicity). The total CPU time needed for the MUPAGE simulation of this data set was 232 hours.

9 Conclusions

In this paper, a fast generator (MUPAGE) of the kinematics of atmospheric muon bundles is presented. As input, it uses parametric formulas for the flux of single and multiple muons valid in the range $1.5 \leq h \leq 5.0$ km w.e. and $\vartheta \leq 85^\circ$. The energy spectrum of single and multiple muons is also simulated, taking into account the dependence of the muon energy on the shower multiplicity and on the distance of the muon from the shower axis. The generator represents an useful tool for underwater/ice neutrino telescope to produce large amount of simulated data. As an example, the generation rate of atmospheric muon bundles on a cylinder with area of 1.4 km^2 laying at a depth of 2475 m and with total energy larger than 3 TeV is 27 Hz. The 8.5×10^8 events corresponding to one year of data, were produced with 232 hours of CPU on 2.33 GHz processor.

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